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Q-ball evolution, B-ball baryogenesis and the f_B problem

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ABSTRACT: We consider some of the issues surrounding the formation and evolution of Q-balls in the MSSM and its extensions. The ratio of the baryon number packed into Q-balls to that outside, f_B, plays a fundamental role in determining the relationship of dark matter to the baryon number in the Universe at present. The final value of f_B will depend upon the details of the formation of Q-balls from the collapse of the Affleck-Dine condensate and upon the subsequent evolution of the ensemble of Q-balls. We discuss the implications for neutralino dark matter in the gravity-mediated scenario and show that a light neutralino is necessary in most cases to account for the baryon to dark matter ratio, with an NMSSM singlino LSP of mass $m_\chi \lesssim 20\,\text{GeV}$ being a favoured candidate.

KEYWORDS: Cosmology of Theories beyond the SM, Dark Matter, Baryogenesis, Supersymmetric Standard Model.

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Contents

1. Introduction

The minimal supersymmetric (SUSY) standard model (MSSM) [1] and its extensions offer a number of possibilities for the origin of the baryon asymmetry. In the MSSM, the most natural possibilities are electroweak baryogenesis [2] and Affleck-Dine (AD) baryogenesis [3]. If we include the possibility of massive Majorana neutrinos we can also have leptogenesis [4].

AD baryogenesis rather generally results in the formation of Q-balls of baryon number (also known as B-balls) in the early Universe [5]–[11]. This occurs because the scalar potential of the squarks generally has attractive interaction terms [12], making the homogeneous Bose condensate of squarks unstable with respect to spatial perturbations. As a result, the condensate fragments to eventually form Q-balls.

In the conventional gravity-mediated MSSM, AD baryogenesis may be characterized by the dimension d of the non-renormalizable terms responsible for lifting the flat directions [13]. For R-parity (R_p) conserving models (necessary to eliminate renormalizable B and L violating operators [1]), the dimension must be even. It has recently been shown that the $d = 4$ models are generally ruled out by the effect of thermal corrections to the AD potential [14], which suppresses the baryon number. Thus $d = 6$ models are favoured. For CP violating phases δ_{CP} (expected to be of the order of 1) these must have a reheating temperature $T_R \approx 1\,\mathrm{GeV}/\delta_{CP}$ in order to account for the observed baryon asymmetry [16]. (Such low reheating temperatures can be a natural feature of SUSY inflation models [15].) The resulting Q-balls are very long lived, typically to temperatures 1MeV – 1 GeV [11]. As a result, it is possible that the late decay of Q-balls to baryons and neutralinos can explain the remarkable similarity of the *number* densities of baryons and dark matter particles for the case of WIMP dark matter with mass $O(m_W)$, for example neutralinos [10, 11]. For $\Omega = 1$ and $\Omega_{DM} \approx 0.4$, the ratio of the number density of baryons to the number density of dark matter particles is given by $n_B/n_{DM} \approx (1.5 - 7.3)\ m_{DM}/m_W$ [17]. (In the following we refer to this as the baryon to dark matter ratio.) This is a major motivation for the idea of late-decaying Q-balls as a simultaneous source of the baryon asymmetry and the dark matter density. More generally, late-decaying Q-balls will provide a source of non-thermal WIMP dark matter, resulting in different predictions for the dark matter density as a function of the parameters of the MSSM [11, 18]. This variant of AD baryogenesis, with late-decaying Q-balls carrying baryon number, is known as B-Ball Baryogenesis [11].

In the case of gauge-mediated SUSY breaking [5]–[9], where Q-ball formation from the AD condensate was first proposed [5]–[7], the potential of the AD scalar becomes essentially completely flat for scalar expectation values larger than the mass of the messenger fields which transmit SUSY breaking [6]. As a result, the mass of the AD scalars in the Q-balls satisfies $m \propto Q^{-1/4}$, and for large enough Q-ball charge the Q-balls are completely stable with respect to decay to nucleons and may account for dark matter [7, 19], with interesting experimental and astrophysical consequences [8]. The cosmology of AD condensate fragmentation in models of gauge-mediated SUSY breaking will be significantly different from the case of gravity-mediated SUSY breaking, but we expect aspects of the gravity-mediated case to be relevant to the gauge-mediated scenario. In this paper we will focus on the more conventional gravity-mediated SUSY breaking scenario.

In both the gravity- and gauge-mediated scenarios the final baryon to dark matter ratio is crucially dependent upon the ratio of baryon number trapped in the Q-balls to that outside, f_B. Both scenarios require that $f_B < 1$. Experimental limits from LEP and Tevatron require that the MSSM lightest SUSY partner (LSP) neutralino mass, m_χ, satisfies $m_\chi > 46\,\mathrm{GeV}$ [20] (this assumes universal A-terms and gaugino masses at the unification scale). As we will show, this implies that $f_B \lesssim 0.4(\Omega_\chi/0.4)$ when the Q-balls decay. In the gauge-mediated scenario, $f_B = 1$ would result in all the baryon number today being in the form of stable Q-balls, an obviously unacceptable scenario.

There have been some attempts to estimate f_B by studying the classical dynamics of the collapse of the AD condensate to non-topological solitions. Enqvist and McDonald [21] considered the collapse of a single spherically symmetric condensate lump. It was observed that the value of f_B depended upon the charge density of the original AD condensate relative to the maximum possible charge, which we denote by Q. As we will show, if Q is less than one then not all the energy in the original AD condensate can be accomodated in the form of positively charged Q-balls. One possibility is that the condensate collapses to higher energy objects, Q-axitons [21], which are classically stable but which can in principle decay to eventually reach a Q-ball. For Q-axitons it was found that f_B could be significantly less than one if $Q \ll 1$, for example $Q = 0.01$ resulted in f_B as low as 0.3 [21]. On the other hand, in a lattice simulation of condensate fragmentation, Kasuya and Kawasaki [23] did not find evidence for f_B significantly less than one. In addition, they also pointed out that it was possible to form positive and negative charged Q-balls, which could in principle allow all the energy in a $Q < 1$ condensate to be entirely accomodated in Q-balls.

In this paper we will discuss various aspects of the f_B problem in more detail. The question of whether $\pm$Q-balls or Q-axitons form will be seen to depend upon the perturbation of the phase of the AD condensate; if the perturbation of the phase of the condensate is larger than the average phase when the condensate fragments, then $\pm$Q-balls will tend to form; if not, then Q-axitons will form. In the case where Q-axitons form there is the queston of how the classically quasi-stable Q-axitons evolve to lower energy Q-balls. We will suggest that this might occur classically via a slow emission of energy in scalar field waves or via annihilations of scalars within the Q-axiton. If the excess energy in the Q-axiton relative to the Q-ball is radiated before the Q-axiton decays and before thermal neutralinos freeze-out then it is possible that f_B can be small enough to account for the baryon to dark matter ratio for a wide range of neutralino masses.

A very different picture emerges if the $Q < 1$ AD condensate collapses to $\pm$Q-balls. In this case, when the Q-balls decay, assuming $f_B \approx 1$ for Q-balls as implied by numerical simulations, the decaying $\pm$Q-balls will produce oppositely charged baryon number but contribute equally to the density of neutralinos, resulting in a large number of neutralinos relative to baryons for $Q \ll 1$. (A similar situation arises if Q-axitons do not lose their excess energy before they decay.) We will show that it is then essential that the MSSM neutralinos annihilate with each other to give an acceptable density. As a result, there will be no direct relationship between the number of baryons and of dark matter particles. However, we will still have a non-thermal relic density of MSSM neutralinos from Q-ball decay, which is an interesting prediction in itself. (There may be some evidence for non-thermal dark matter from non-singular galactic halos [24].)

If we have a maximally charged AD condensate, $Q = 1$, then there will be direct formation of Q-balls without a Q-axiton stage. In this case $f_B \approx 1$ and MSSM neutralinos consistent with experimental constraints cannot account for dark matter directly via Q-ball decay. Annihilations must reduce the number of neutralinos, again losing the direct connection with the baryon number density although still producing non-thermal neutralino dark matter.

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Thus it will be seen that it is generally difficult for the baryon to dark matter ratio to be explained via Q-ball decay in the context of the MSSM, with the only possible exception being the limiting case where Q-axitons form and evolve to Q-balls before neutralinos freeze out. However, extensions of the MSSM with weaker lower bounds on the LSP mass, in particular the next-to-minimal SUSY standard model (NMSSM) [25] with light singlino LSPs [26]–[28], will be seen to be consistent with the baryon to dark matter ratio.

The paper is organized as follows. In section 2 we consider the classical value of f_B from condensate fragmentation. We consider the initial conditions for the linear evolution of the perturbations and the conditions for the formation of Q-axitons versus ±Q-balls. We then comment on the numerical simulations of condensate fragmentation and point out that a sufficiently long evolution will be necessary to reach the time when absorption/re-emission of scalar field waves from collapsing condensate lumps becomes negligible and the condensate lump can reach their equilibrium states, allowing f_B to be extracted. In section 3 we discuss the possible evolution of a Q-axiton to a stable Q-ball. In section 4 we consider the consequences for neutralino dark matter in the MSSM and the NMSSM. In section 5 we comment on the effects of more complicated aspects of realistic condensate fragmentation on the value of f_B. In section 6 we present our conclusions.

2. Classical evolution of the AD condensate

In this section we consider some aspects of the classical evolution of perturbations. The formation of Q-axitons versus ±Q-balls will depend upon the initial size of the perturbation in the phase of the AD condensate when coherent oscillations begin and the baryon asymmetry is established at $H \approx m$, where m is the mass of the AD scalars.

2.1 Energy in Q-balls for a non-maximally charged condensate

We first show that for a non-maximally charged condensate in the gravity-mediated scenario it is not possible for all the energy in the condensate to be accomodated in positively charged Q-balls. The energy density in a coherently oscillating scalar field condensate with amplitude ϕ_o is $\rho = m^2\phi_o^2$. The maximum possible charge density in the condensate is $\rho_{Q\,\max} = \rho/m \equiv m\phi_o^2$, corresponding to the case where all the scalars in the condensate are charged. The maximum energy density in Q-balls is $\rho_{q-balls} = m\;\rho_Q = mQ\;\rho_{Q\,\max} = Q\rho$, since the mass of the scalars in the Q-balls is to a good approximation m in the gravity-mediated scenario [11] and we assume that all the charge is packed into Q-balls. So the energy density in Q-balls to the energy density in the condensate is

$$\frac{\rho_{q-\mathrm{ball}}}{\rho} = Q\,. \qquad (2.1)$$

Thus for a non-maximally charged condensate with $Q < 1$ not all the energy density can be accomodated in positively charged Q-balls. One must either form higher energy non-topological solitons (Q-axitons) or have an ensemble of positively and negatively charged Q-balls.

2.2 Conditions for Q-axiton versus ±Q-ball formation

We next review the evolution of perturbations at $H > m$ [21]. Perturbations of the AD condensate may be expected to arise from quantum fluctuations of the AD scalar during inflation, with $\delta\phi \approx H/2\pi$. The subsequent evolution of the perturbations will depend upon the paramaters of the scalar potential. In general, the perturbations satisfy an equation of the form [21]

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2\delta\phi = -kH^2\delta\phi\,, \tag{2.2}$$

where k is determined by the parameters c, $|a_\lambda|$ and d of the AD potential [21],

$$U(\Phi) \approx (m^2 - cH^2)\left(1 + K\log\left(\frac{|\Phi|^2}{M^2}\right)\right)|\Phi|^2 + \frac{\lambda^2|\Phi|^{2(d-1)}}{M_*^{2(d-3)}} + \left(\frac{A_\lambda\lambda\Phi^d}{dM_*^{d-3}} + \mathrm{h.c.}\right). \tag{2.3}$$

Here $K < 0$ is due to radiative corrections (with $|K| \approx 0.01 - 0.1$) [10]–[12], d is the dimension of the non-renormalizable term in the superpotential which lifts the flat direction (with $d = 6$ favoured), cH^2 gives the order H^2 correction to the scalar mass [29] (with c positive and typically of the order of one for AD scalars) and we assume that the natural scale of the non-renormalizable terms is the supergravity mass scale $M_* = M_{Pl}/\sqrt{8\pi}$ [1]. The A-term also receives order H corrections, $A_\lambda = A_{\lambda\,o} + a_\lambda H$, where $A_{\lambda\,o}$ is the gravity-mediated soft SUSY breaking term and a_λ depends on the nature of the inflation model; for F-term inflation $|a_\lambda|$ is typically of the order of one [13] whilst for minimal D-term inflation models it is zero [30].

For the case of the AD scalar, $\Phi = (\phi_1 + i\phi_2)/\sqrt{2}$, we find different values for k, k_1 and k_2, corresponding to the equations of motion for $\delta\phi_1$ and $\delta\phi_2$ [21]. Thus the evolution of the $\delta\phi_1$ and $\delta\phi_2$ perturbations can be quite different. As a result, the initial $\delta\phi_1$ and $\delta\phi_2$ at $H \approx m$ depends upon the parameters of the potential. In particular, if $|a_\lambda|$ is larger than $|c|$, then k_2 can be large compared with k_1, which means that $\delta\phi_2$ can be much more suppressed by expansion than $\delta\phi_1$ [21]. (During matter domination $\delta\phi \propto a^{-Re(\eta)}$, where $\eta = (3/2 - \sqrt{9/4 - 4k})/2$, whilst during inflation $\delta\phi \propto a^{-Re(\sigma)}$, where $\sigma = (3 - \sqrt{9 - 4k})/2$ [21].) Thus depending on $|c|$ and $|a_\lambda|$, $\delta\phi_1 \gg \delta\phi_2$ or $\delta\phi_1 \ll \delta\phi_2$ at $H \approx m$ are possible.

As H becomes smaller than m, the A-term becomes dominated by $A_{\lambda\,o}$. The real direction is determined by the A-term, so if we define ϕ_1 and ϕ_2 relative to the real direction then these fields are rotated at $H < m$ relative to their directions at $H > m$. (At $H > m$, ϕ_2 is damped to zero due to the A-term in the potential [21].) In addition, due to the A-term, the coherent oscillations of ϕ_1 and ϕ_2 will have a relative phase shift typically of the order of 1, resulting in a baryon asymmetry approximately proportional to the phase $\theta = \phi_{2\,o}/\phi_{1\,o}$ of the condensate field, where $\phi_{1\,o}$ and $\phi_{2\,o}$ are the amplitudes of the coherent oscillations. If the θ between is small compared with one and $\delta\phi_1 \gg \delta\phi_2$ at $H \gtrsim m$, then $\delta\phi_2$ at $H \approx m$ after this rotation will be given by $\delta\phi_2 \approx \theta\delta\phi_1$. This is the smallest initial value of the $\delta\phi_2$ relative to $\delta\phi_1$ possible at $H \approx m$. This limiting case is relevant to the possibility of forming Q-axitons rather than $\pm$Q-balls and we will focus on it in the following.

We can then consider the growth of perturbation until they become non-linear. This has been studied analytically for the case of a maximally charged (MAX) $Q = 1$ AD condensate [7, 11, 21]. However, it has not been considered in detail for the case of a non-MAX condensate, for which the amplitude of ϕ_1 is large compared with that of ϕ_2. The equations of motion are

$$\ddot{\phi}_i + 3H\dot{\phi}_i - \nabla^2\phi_i = -m^2(1+K)\phi_i - Km^2\phi_i log\left(\frac{\phi_1^2+\phi_2^2}{\phi_o^2}\right), \qquad i = 1, 2\,. \quad (2.4)$$

For $\phi_1(t) \gg \phi_2(t)$ during most of the oscillation cycle, the equation for ϕ_1 will be similar to the case of the MAX condensate, except that $\phi_1^2 + \phi_2^2$ will be varying periodically in time rather than constant. Thus the equation for the growth of space-dependent perturbations of ϕ_1 will be similar to the MAX condensate and we expect that the ϕ_1 perturbations will go non-linear ($\delta\phi_1/\phi_1 \gtrsim 1$) essentially as in the case of the MAX condensate [7, 11]. The equation for perturbations in ϕ_2 will, however, be different, since the *log* term is dominated by ϕ_1^2. Even if $\delta\phi_2 > \phi_2(t)$, the equation for perturbations of $\phi_1 (\equiv \phi_1(t) + \delta\phi_1)$ is not significantly altered so long as $\delta\phi_2/\phi_1 < 1$. Thus the perturbation in the energy density (which is essentially determined by ϕ_1) will not go non-linear until $\delta\phi_1/\phi_1 \gtrsim 1$, even if $\delta\phi_2/\phi_2 \gg 1$.

We next show that $\delta\phi_2/\phi_1 \lesssim \delta\phi_1/\phi_1$ at all times if $\delta\phi_2 \lesssim \delta\phi_1$ initially at $H \approx m$. To prove this we must show that the rate of growth of $\delta\phi_2$ is no greater than that of $\delta\phi_1$. We consider $\phi_1 \gg \phi_2$, as is true for most of the oscillation cycle, and expand the equations of motion,

$$\delta\ddot{\phi}_1 + 3H\delta\dot{\phi}_1 - \nabla^2\delta\phi_1 = -m^2(1+3K)\delta\phi_1 - 2Km^2\phi_2\frac{\delta\phi_2}{\phi_1}\,, \quad (2.5)$$

$$\delta\ddot{\phi}_2 + 3H\delta\dot{\phi}_2 - \nabla^2\delta\phi_2 = -m^2(1+K)\delta\phi_2 - 2Km^2\phi_2\frac{\delta\phi_1}{\phi_1}\,. \quad (2.6)$$

Since $\theta\delta\phi_1 = (\phi_2/\phi_1)\delta\phi_1 \lesssim \delta\phi_2$, the magnitude of the $|K|$ dependent term of the $\delta\phi_2$ equation, which is responsible for the growth of the $\delta\phi_2$ perturbations, is less than that of the $\delta\phi_1$ equation. Thus if $\delta\phi_2 \lesssim \delta\phi_1$ initially at $H \approx m$ then at all times $\delta\phi_2 \lesssim \delta\phi_1$. In particular, $\delta\phi_2 \approx \theta\delta\phi_1$ is a consistent soultion of the equations. Therefore for $\delta\phi_2 \lesssim \delta\phi_1$ at the onset of coherent oscillations, non-linearity will occur only once $\delta\phi_1$ goes non-linear, at which point the condensate will begin to fragment to condensate lumps.

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In general this means that $\delta\phi_2/\phi_2 \gg \theta$ is possible when the energy density in condensate goes non-linear ($\delta\phi_1 \approx \phi_1$) and the condensate fragments i.e. the fluctuation in the phase of the condensate can be large compared with the average phase. In this case we expect ±Q-balls to form, since the charge density of the non-MAX condensate is proportional to $\phi_1\phi_2 \approx \phi_1\delta\phi_2$ and $\delta\phi_2$ is periodic in space with positive and negative values. However, in the limiting case where $\delta\phi_2 \approx \theta\delta\phi_1$ at $H \approx m$ (and θ is small compared with 1), $\delta\phi_2 \approx \theta\delta\phi_1$ is a consistent solution throughout. Therefore when the condensate fragments we will have $\delta\phi_2/\phi_2 \approx 1$ i.e. the fluctuation of the phase of the condensate is approximately the same as the average phase. In this limiting case we expect that Q-axiton formation will be favoured over ±Q-ball formation. A full numerical evolution of the perturbations will be necessary to determine exactly what happens in this case, but we expect that there will be dominant formation of Q-axitons together with a few ±Q-balls.

So the question of whether Q-axitons or ±Q-balls form will be detemined by the fluctuation in the phase at the onset of condensate fragmentation. This will all depend upon the evolution of the perturbations after they leave the horizon during inflation, which in turn depends on the parameters of the AD scalar potential. We believe that both cases are possible, with ±Q-ball formation being typical but with Q-axiton formation as a limiting case.

2.3 Numerical simulations and absorption/re-emission of scalar waves

We next consider the classical evolution of the non-linear condensate lumps. For the case of Q-balls (MAX condensate or non-MAX with ±Q-balls), this is straightforward. Non-relativistic Q-balls carrying essentially all the charge form directly from the condensate and move apart from each other due to the expansion of the Universe until their decay rate to quarks is fast enough relative to the expansion rate to allow them to evaporate away into baryons and neutralinos [11]. For the case of a non-MAX condensate with Q-axitons, the non-linear evolution of a single spherically symmetric condensate lump appears to show that $f_B < 1$ once the lump reaches the classically stable Q-axiton state [21]. It takes some time to reach the Q-axiton state, via emission of scalar field waves from a pulsating condensate lump [21]. However, in realistic models we cannot consider a single, isolated condensate lump, but instead must consider an ensemble of lumps distributed throughout space. This leads to the possibility that scalar waves emitted by one relaxing condensate lump might be absorbed by a neighbouring condensate lump (since the phase of the Q-axitons (ranging roughly from 0 to 2θ) on average will be the same as the phase θ of the initial condensate lump, absorption and emission of scalar field waves should be relatively efficient and without reflection effects due to large (order π) phase differences [31]). As a result it is possible that the condensate lumps are maintained in an excited state until expansion pulls them far enough apart for absorption/re-emission to be no longer effective. Only then can the condensate lumps reach their classically stable Q-axiton state and f_B be extracted from lattice simulations [23] of condensate collapse.

The condition for absorption/re-emission to be important can be estimated as follows. We can consider a "lattice" of condensate lumps to form just after condensate collapse. These lumps will pulsate about their stable Q-axiton configuration, emitting scalar field waves as they do [21]. For simplicity, we will replace the discrete condensate lumps with a continuous number density, in order to estimate the number of condensate lumps encountered by an expanding scalar wavefront. Suppose the condensate collapses to form condensate lumps at t_o, H_o. Consider a scalar field pulse emitted by a lump at a later time t_i. The condensate lump density is

$$n_Q(t) = n_{Q\,o} \left(\frac{a_o}{a}\right)^3 , \tag{2.7}$$

where $n_{Q\,o} = n_Q(t_o)$. The charge absorbed by the surrounding condensate lumps may be estimated by considering the area of the outgoing wave removed by encountering other lumps. The lumps have area

$$A_l \approx 4\pi r_l^2 , \tag{2.8}$$

where r_l is the radius of the lump. The charge density of the outgoing wavefront of radius r is

$$\sigma_Q = \frac{Q_W}{4\pi r^2} , \tag{2.9}$$

where Q_W is the total charge carried by the wavefront. Thus the charge lost upon encountering a condensate lump is $\sigma_Q A_l$. The charge absorbed from a wave emitted at t_i is therefore

$$\Delta Q = \int_{t_i}^{t} n_Q(t) \sigma_Q A_l 4\pi r^2(t) v(t) dt = n_{Q\,i} Q_W A_l v_o H_i^{-1} \int_{a_i}^{a} \left(\frac{a_i}{a}\right)^{7/2} a_i^{-1} da , \tag{2.10}$$

where the velocity $v(t)$ of the scalar field waves is initially set by a wavelength of the the order of the Q-axiton radius, corresponding to momentum $k \approx 2\pi |K|^{1/2} m$ [21], which implies that the scalar field waves are initially mildly relativistic, $v(t_i) = v_o \approx 1$, and that subsequently $v = v_o(a_i/a)$. Therefore the charge absorbed is

$$\frac{\Delta Q}{Q_W} = \frac{2}{5} \frac{n_{Q\,i} A_l v_o}{H_i} \left(1 - \left(\frac{a_i}{a}\right)^{5/2}\right) . \tag{2.11}$$

Thus for $a_i/a > 1$ the charge absorbed is given by

$$\frac{\Delta Q}{Q_W} \approx \frac{2 A_l n_{Q\,i} v_o}{5 H_i} . \tag{2.12}$$

The condition for total absorption of the outgoing wave is $(\Delta Q/Q_W) > 1$. In terms of H_o and a_o, $H_i = H_o (a_o/a_i)^{3/2}$. When the condensate lumps form, the radius of the spherically symmetric condensate lumps is approximately the same as the spacing between the lumps, $r_l \approx (|K|^{1/2}m)^{-1}$ [11, 21]. Condensate lumps form at $H_o \approx (2|K|m)/\alpha(\lambda_o)$ where $\alpha(\lambda_o) \approx 30$ [11]. Thus from eq. (2.12) the condition for absorption by condensate lumps to be negligible is that the expansion factor must satisfy $a > a_c$, where

$$\frac{a_c}{a_o} = \left(\frac{3v_o}{5}\right)^{2/3} \frac{\alpha^{2/3}}{|K|^{1/3}} . \tag{2.13}$$

This must be satisfied in order to be certain that a numerical simulation of condensate fragmentation has allowed the condensate lumps to reach their Q-axiton equilibrium configuration, in which case f_B can be extracted. In a recent simulation [23] of condensate fragmentation on the lattice, Kasuya and Kawasaki found that generally $f_B = 1$ to a good approximation. Their simulation runs from initial time $t_o = 5 \times 10^3 m^{-1}$ to $t_f = 10^5 m^{-1}$ and for values of $|K| = 0.01,\ 0.05,\ 0.1$. The scale factor at the end of their simulation, a_f, is thus

$$\frac{a_f}{a_o} = \frac{(1 + (3/2)H_o t_f)^{2/3}}{(1 + (3/2)H_o t_o)^{2/3}} \approx \left(\frac{t_f}{t_i}\right)^{2/3} = 6.9 . \tag{2.14}$$

On the other hand, the values of a_c/a_o for $|K| = (0.01,\ 0.05,\ 0.1)$ are (31.7, 18.5, 14.3). Thus the simulation is well within the time scale during which absorption/re-emission will be efficient. This might account for the large value of f_B obtained in [23] as compared with isolated spherically symmetric lump evolution. In order to ascertain whether $f_B = 1$ is correct, or whether a $|K|$ and Q dependent value of f_B can occur as suggested by single lump evolution [21], a longer simulation may be necessary.

The simulation in [23] also shows that for smaller angular velocities of the complex condensate field (i.e. amplitude of ϕ_2 small compared with ϕ_1), $\pm$Q-ball formation becomes important. This is consistent with fluctuations in the phase becoming larger than the average phase (ϕ_2/ϕ_1) when the condensate fragments, assuming that the initial perturbation of the condensate is unaltered as the angular velocity (ϕ_2) is reduced.

3. Evolution of the Q-axiton

We next consider if and how a Q-axiton relaxes to the lower energy stable Q-ball. In numerical simulations [21] the Q-axiton is stable on time scales much larger than m^{-1}; the attractive force between the scalars in the condensate is balanced by the gradient energy.[1] On the other hand, it has been shown that there is no stable spherically

[1]Somewhat similar long-lived "pseudo-breather states" are also observed in $3+1$ Sine-Gordon theory, which may represent long-lived pionic states that could be produced in heavy-ion collisions [32].

symmetric breather soliton (such as the Q-axiton) in $3+1$ dimensions [22]. Thus we expect that the Q-axiton will evolve to a lower energy state, most likely a Q-ball. It is not apparent from the existing numerical simulation how it loses energy, but from the equations of motion we can get some idea of the process. In the equations of motion for ϕ_1 and ϕ_2 the argument of the *log* term, eq. (2.4), $\phi_1^2 + \phi_2^2$, is varying periodically in time. This is contrast with the case of the Q-ball, for which the amplitudes of ϕ_1 and ϕ_2 are the same such that $\phi_1^2 + \phi_2^2$ is time-independent. It is this property that allows one to factor out the time dependence in the Q-ball solution, $\Phi(r,t) \sim \phi(r)e^{i\omega t}$. For the Q-axiton we cannot factor out the time dependence, so we expect the magnitude of the Q-axiton field to be varying periodically in time on a time scale $\approx m^{-1}$, albeit very slightly based on the numerical results [21]. Thus we expect to find that the Q-axiton has a "quivering" solution. This will then result in the emission of scalar field waves carrying energy and perhaps charge. The Q-axiton could then either evolve to a Q-ball of the same charge or could even, in the opposite limit, completely evaporate away. However, since the rate at which energy is being lost by the Q-axiton is very small on the dynamical time scale of the scalar field, m^{-1}, and since the lowest possible energy final state would be to have all the charge in the form of a Q-ball, it is likely that this will be the final state of the process. If this picture is correct then although the Q-axiton appears to be stable on relatively long time scales compared with m^{-1} [21], on time scales corresponding to the inverse Hubble parameter at the time of Q-ball decay it is likely that the Q-axiton will have evolved to the corresponding Q-ball. It remains to be shown that Q-axitons do indeed evolve to Q-balls and that the time scale for Q-axiton evolution is indeed smaller than that of Q-ball decay.

The above process is purely classical in nature. It is also possible that the excess energy in the Q-axiton could be released via annihilations of the scalars. We can consider the Q-axiton to be, to a first approximation, a superposition of two coherently oscillating scalar fields ϕ_1 and ϕ_2. The annihilation rate is $n_i^{-1} dn_i/dt \propto n_i \;\; (i = 1, 2)$. Therefore since $n_1 \approx m\phi_1^2 \gg n_2 \approx m\phi_2^2$, the rate at which the ϕ_1 amplitude decreases will be much larger than that of the ϕ_2 amplitude. Therefore $\phi_{1\,o} \rightarrow \phi_{2\,o}$ i.e. the Q-axiton tends towards a Q-ball. The final Q-ball charge to the initial Q-axiton charge will be $Q_f/Q_i = \phi_{2\,f}/\phi_{1\,i} \leq \phi_{2\,i}/\phi_{1\,i} = Q$, the charge of the initial AD condensate. Thus if annihilations are able to allow the Q-axiton to evolve to a Q-ball before the Q-axiton decays and before the neutralinos freeze out of chemical equilibrium, then $f_B \rightarrow Q\; f_{B\,cl}$, where $f_{B\,cl}$ is the classical value of f_B for the Q-axiton.

We next consider the decay and annihilation of the scalars in the Q-axiton. Q-balls and Q-axitons from AD condensate fragmentation have a thick wall profile, well-described by a Gaussian profile $\phi(r) = \phi(0)e^{-r^2/R^2}$, where $R \approx (|K|^{1/2}m)^{-1}$ is the Q-ball/Q-axiton radius [11, 23]. Q-ball decay was originally analysed for the case of a thin-wall Q-ball with scalars decaying to fermion pairs [33]. The decay rate was found to be

$$\left(\frac{dB}{dt}\right)_{\rm fermion} \lesssim \frac{\omega^3 A}{192\pi^2}\,, \tag{3.1}$$

where $\omega \approx m$ and A is the area of the Q-ball. This is based on scalar decay filling the phase space for fermions of energy $m/2$ throughout the Q-ball, resulting in Pauli blocking of further decay and a Q-ball decay rate which is proportional to A since fermions can escape at the surface. In the case of the thick walled Q-ball in the MSSM there is no equivalent estimate of the decay rate to fermion pairs. However, we expect the order of magnitude to be similar to the thin-wall case for a given $\phi(0)$, since the decay rate is simply determined by completely filling the fermion phase space within the Q-ball volume. In addition, in the case of Q-balls in the MSSM it is possible to have decays to pairs of scalars and gauge bosons, which are not suppressed by Pauli blocking. In [11] we considered the decay rate to scalars for the case of a thick walled Q-ball. It was shown that most of the Q-ball decay came from scalar decays occuring in a region of width $\delta r \approx R^2/4r_*$ around $r_* = \gamma R$ (where $\gamma = ln^{1/2}(g\phi(0)/m) \approx 5$), where tree level decay to pairs of particles coupling to ϕ, of mass $g\phi(r)$, becomes kinematically possible. The resulting Q-ball decay rate was found to be enhanced by a factor up to 10^3 compared with the thin-wall fermion decay rate, but is still proportional to A.

We assume that the AD scalar decay rate to pairs of bosons or fermions has the form

$$\begin{aligned}\Gamma(r) &\approx \frac{\alpha^2 m^3}{\phi^2}\,, \qquad g\phi > m \\ &\approx \alpha m\,, \qquad g\phi < m\,,\end{aligned} \tag{3.2}$$

(which assumes the largest possible decay rate via exchange of heavy particles once $g\phi > m$). We first show that in general Pauli blocking/Bose enhancement effects are likely to be significant in Q-ball decay. The maximum number density of particles before the occupation number of states becomes greater than 1 is given by

$$n_{\rm max} \approx \frac{p^2 \Delta p}{\pi^2}\,, \tag{3.3}$$

where $p \equiv |\mathbf{p}| = m/2$ is the momentum of the final state particles and Δp is the spread in momentum of the final state particles. Since the AD scalars in the Q-axiton are confined to be within a radius R^{-1} we expect $\Delta p \approx R^{-1}$ and so

$$n_{\rm max} \approx \frac{|K|^{1/2} m^3}{4\pi^2}\,. \tag{3.4}$$

Pauli blocking/Bose enhancement effects will be important if the density of particles from AD scalar decays is greater than $n_{\rm max}$. Relativistic particles can escape from the region of the thick-walled Q-ball δr around r_*, where most of the decays occur, in a time $\tau \approx \delta r \approx R/20$. Thus the number density of particles from AD scalar decay is

$$n_{\text{decay}} \approx n\Gamma_{\text{decay}}\tau \approx \frac{4\pi m^3}{20|K|^{1/2}}\,, \tag{3.5}$$

where we have used $n \approx m\phi^2(r)$ and $\phi^2(r_*) \approx m^2/g^2$. Therefore

$$\frac{n_{\text{decay}}}{n_{\text{max}}} \approx \frac{4\pi^3}{5|K|}\,. \tag{3.6}$$

This is typically large compared with 1, since $|K| \approx 0.01 - 0.1$ in the MSSM [12]. Therefore Pauli blocking or Bose enhancement effects are likely to be significant in the decay of AD scalars in the Q-axiton. The result of this is that all the scalars in the region δr around r_* will rapidly decay. These will be replenished by the Q-axiton reconfiguring itself on the dynamical time scale m^{-1}. In the absence of Bose enhanced decay, in a time m^{-1} the fraction of the scalars in the region δr which decay is $\Gamma_{\text{decay}} m^{-1} \approx \alpha$. Therefore the enhancement in the decay rate is of the order of α^{-1}, which will increase the decay temperature of the Q-axiton by $\alpha^{-1/2}$.

We next compare the decay and annihilation rates. Most of the decays occur around $r \approx r_*$ via tree level decays of the AD scalars [11]. We first show that AD scalars in the Q-axiton can annihilate at tree-level to pairs of gauge bosons but cannot decay to pairs of gauge bosons. To do this we consider a simple $U(1)$ gauge interaction with a pair of "quark" superfields Q_1 and Q_{-1}, with $U(1)$ charges ± 1. The terms of the Lagrangian associated with the gauge interaction are then

$$\begin{aligned}\mathcal{L} = {} & D_\mu Q_1^\dagger D^\mu Q_1 + D_\mu Q_{-1}^\dagger D^\mu Q_{-1} + (igQ_1^\dagger\overline{\lambda}\psi_{Q_1} - igQ_{-1}^\dagger\overline{\lambda}\psi_{Q_{-1}} + \text{h.c}) + \\ & + \frac{g^2}{2}\left(|Q_1|^2 - |Q_{-1}|^2\right)^2 .\end{aligned} \tag{3.7}$$

We can rewrite this in terms of a flat direction scalar Φ and its orthogonal partner Θ,

$$\Phi = \frac{1}{\sqrt{2}}(Q_1 + Q_{-1})\,, \tag{3.8}$$

$$\Theta = \frac{1}{\sqrt{2}}(-Q_1 + Q_{-1})\,. \tag{3.9}$$

The kinetic terms in $\mathcal{L}$ are then

$$(\partial_\mu\Phi)^\dagger\partial^\mu\Phi + (\partial_\mu\Theta)^\dagger\partial^\mu\Theta + g^2A^\mu A_\mu(\Phi^\dagger\Phi + \Theta^\dagger\Theta) - gA^\mu(i\Phi\partial_\mu\Theta^\dagger + i\Theta\partial_\mu\Phi^\dagger + \text{h.c.})\,. \tag{3.10}$$

From this we see that on introducing an expectation value for $Re(\Phi)$, representing the Q-axiton field, the Goldstone boson corresponds to $Im(\Theta)$. Thus pairs of Q-axiton scalars couple to the massive gauge bosons but there is no linear coupling of $Re(\Phi)$ to gauge bosons once the Goldstone boson is rotated away. As a result, we can have Q-axiton scalar annihilations to pairs of gauge boson but no decays to pairs of gauge bosons. The scalars can, however, decay to gaugino-quark pairs via the interaction

$$igQ_1\overline{\psi}_{Q_1}\lambda - igQ_{-1}\overline{\psi}_{Q_{-1}}\lambda + \text{h.c.} \to -ig\Phi\overline{\psi}_{\Theta}\lambda + \text{h.c.}\,, \tag{3.11}$$

with decay rate

$$\Gamma_{\text{decay}} \approx \alpha m\,, \tag{3.12}$$

where $\alpha = g^2/4\pi$. Pairs of AD scalars can annihilate to gauge boson pairs, with annihilation cross-section

$$\sigma_{\text{ann}} \approx \frac{\alpha^2}{m^2}\,. \tag{3.13}$$

Thus the annihilation rate is

$$\Gamma_{\text{ann}} = n\sigma_{\text{ann}} \approx m\phi(r^*)^2\sigma_{\text{ann}} \approx \frac{\alpha m}{4\pi}\,, \tag{3.14}$$

where n is the number density of AD scalars. From this we expect that the annihilation rate to gauge bosons will be no larger than the decay rate to gaugino-squark pairs. The only way to have a significant suppression of the decay rate relative to the annihilation rate would be if the decay to gauginos was kinematically suppressed such that the decay occurs only to quarks and squarks via small Yukawa couplings. This would depend on the mass of the AD scalar relative to the gauginos and the generational structure of the mixture of squarks forming the AD scalar. Bose enhancement of the annihilation rate to gauge bosons and Pauli blocking of decay rate to gauginos could also help to enhance annihilations relative to decays.

Therefore it is possible that in the limiting case where the non-MAX condensate fragments to Q-axitons rather than ±Q-balls, the Q-axiton can evolve to a Q-ball before the Q-balls decay, either by classical evolution from the quasi-stable Q-axiton state or, less likely, by scalar annihilations dominating over decays. The value of f_B when the Q-balls decay is particularly small in the annihilation case, which could be important in allowing a wide range of neutralino masses to be consistent with the baryon to dark matter ratio. However, a more precise analytical and numerical study will be required to show exactly how the Q-axiton evolves to a Q-ball.

4. Consequences for the dark matter and the baryon to dark matter ratio

4.1 Neutralinos directly from Q-ball decay

As discussed in the Introduction, a major motivation for B-ball Baryogenesis is the similarity, to within an order of magnitude, of the *number* densities of baryon and dark matter particles for the case where dark matter particles have masses of the order of m_W. Q-balls from a $d = 6$ AD condensate are expected to decay at a temperature $T_d \approx 1\text{MeV} - 1\,\text{GeV}$ [11]. Since the scalars in the Q-ball (or Q-axiton) are R_p-odd (essentially squarks and sleptons) they will produce one LSP neutralino per scalar. For the case of Q-balls from a MAX condensate, assuming that there is no subsequent annihilation of the neutralinos from Q-ball decay, the number density of neutralinos will be related to the baryon number density by

$$n_\chi = 3 f_B n_B \,, \tag{4.1}$$

or equivalently

$$\Omega_\chi = 3 f_B \left(\frac{m_\chi}{m_n}\right) \Omega_B \,. \tag{4.2}$$

Nucleosynthesis constrains Ω_B; we consider two possible ranges,

$$0.0048 \lesssim \Omega_B h^2 \lesssim 0.013 \tag{4.3}$$

from "reasonable" bounds on primordial element abundances [11, 34] and

$$0.004 \lesssim \Omega_B h^2 \lesssim 0.023 \tag{4.4}$$

from conservative bounds [23, 35]. With $h = 0.6 - 0.8$ [36, 17] we obtain upper bounds on the LSP mass neutralino:

$$m_\chi < 17.6\, f_B^{-1} \left(\frac{\Omega_\chi}{0.4}\right) \left(\frac{h}{0.8}\right)^2 \text{GeV} \tag{4.5}$$

from the reasonable nucleosynthesis bounds and

$$m_\chi < 20.8\, f_B^{-1} \left(\frac{\Omega_\chi}{0.4}\right) \left(\frac{h}{0.8}\right)^2 \text{GeV} \tag{4.6}$$

from the conservative bounds.

For the case of the MSSM, present experimental bounds on the LSP neutralino mass depend upon the theoretical assumptions made. The direct experimental lower limit on the neutralino mass from ALEPH is $m_\chi > 32.3\,\text{GeV}$ [37], which requires that $f_B \lesssim 0.64$, assuming the conservative nucleosynthesis limit. If one evolves the renormalization group equations assuming universal A-terms and gaugino masses at the unification scale then LEP and Tevatron constraints imply that $m_\chi > 46\,\text{GeV}$ ($m_\chi > 51\,\text{GeV}$ with universal scalar masses also) [20], which requires that $f_B \lesssim 0.45$ ($f_B \lesssim 0.41$ with universal scalar masses), assuming the conservative nucleosynthesis limits. Thus the case of Q-balls with $f_B \approx 1$ (expected for the MAX condensate), is not compatible with MSSM neutralinos.

If we consider a non-MAX condensate then we can either form $\pm$Q-balls or, as a limiting case, Q-axitons. If the Q-axitons cannot evolve to Q-balls before the neutralinos freeze-out then the extra energy in the Q-axiton relative to a Q-ball of the same charge will result in more neutralinos being produced in the decay, such that $f_B \rightarrow f_B/Q$ in the m_χ upper bounds. The same correction is necessary in $\pm$Q-ball decay. (So as far as the baryon to dark matter ratio is concerned there is no real difference between Q-axitons and $\pm$Q-balls in this case.) So for $Q < 1$ the constraints on f_B are correspondingly tighter, further disfavouring Q-ball decay as an explaination of the baryon to dark matter ratio in the MSSM. (For the $\pm$Q-ball case, $f_B = 1$ is expected, whereas for the Q-axiton f_B is estimated to be as small as 0.3 for $Q \approx 0.01$ [21]. However, even in this case f_B/Q is too large to be compatible with MSSM neutralinos.)

Thus the only way the MSSM can be compatible with direct Q-ball decay to neutralinos as an explaination of the baryon to dark matter ratio is if we have the limiting case where Q-axitons form from a non-MAX condensate and evolve to Q-balls before the neutralinos freeze-out. In this case, for small condensate charge Q, f_B could be small enough to allow dark matter MSSM neutralinos compatible with experiment to come directly from Q-ball decay, especially in the case where annihilations are responsible for the evolution of the Q-axitons to Q-balls.

If we take the view that the baryon to dark matter ratio is indeed due to decaying Q-balls then, modulo the possibility of Q-axiton formation and evolution, we must consider an alternative to MSSM LSP neutralinos. A natural possibility is to consider the NMSSM with a mostly singlino LSP [26]–[28]. With $f_B \approx 1$ and $Q \leq 1$, Q-ball decay imples that the LSP must have a mass less than about 20 GeV. Current experimental bounds combined with renormalization group evolution indicate a lower bound of 3-5 GeV on the singlino LSP mass [28] (singlinos may be excluded by vacuum stability considerations, but this depends on the universality conditions at the unification scale [27]).

So, given that $f_B/Q \gtrsim 1$ is likely when the Q-balls decay, Q-ball decay combined with the baryon to dark matter number ratio and experimental bounds on neutralino masses rules out the MSSM but is compatible with the NMSSM with a mostly singlino LSP of mass less than about 20 GeV. This may be regarded as a prediction of the B-ball Baryogenesis explaination of the baryon to dark matter ratio.

4.2 Effect of annihilations

In the above it has been assumed that the neutralinos from Q-ball decay do not annihilate. However, if a large enough density of neutralinos is produced in Q-ball decay, they will annihilate with each other. In this case the number density of LSP neutralinos is determined by the annihilation cross-section and is not directly related to the baryon number density. It will also be spatially constant, ruling out the possibility of neutralino isocurvature fluctuations correlated with the baryon number [38].

The upper limit on the number of LSPs at a given temperature is [11]

$$n_{\text{limit}}(T) = \left(\frac{H}{< \sigma v >_{\text{ann}}}\right)_T , \tag{4.7}$$

where σ is the annihilation cross-section, v is the relative velocity of the neutralinos and $\langle \ldots \rangle$ denotes the thermal average. For neutralinos $\langle \sigma v \rangle_{\text{ann}} = a + bT/m_\chi$, where a and b are determined by the parameters of the SUSY model [39]. For the case of light neutralinos $m_\chi < m_W$, the a term is negligible and the annihilation cross-section is b dominated, in which case $n_{\text{limit}} \propto g(T)^{1/2} T$. Therefore

$$n_{\text{limit}}(T) = \left(\frac{g(T_f)}{g(T)}\right)^{1/2} \left(\frac{T_f}{T}\right)^3 n_{\text{relic}}(T) , \tag{4.8}$$

where

$$n_{\text{relic}}(T) = \left(\frac{g(T)}{g(T_f)}\right) \left(\frac{T}{T_f}\right)^3 \left(\frac{H}{< \sigma v >_{\text{ann}}}\right)_{T_f} \tag{4.9}$$

is the thermal relic neutralino density and T_f is the neutralino freeze-out temperature (typically $T_f \approx m_\chi/20$). Thus in order to account for the baryon to dark matter ratio the LSP density from Q-ball decay must satisfy

$$n_{relic}(T_d) < n_{LSP}(T_d) < n_{\text{limit}}(T_d) . \tag{4.10}$$

In general

$$n_{LSP}(T_d) = \frac{3 f_B n_B}{Q} . \tag{4.11}$$

Thus the condition $n_{LSP}(T_d) < n_{\text{limit}}(T_d)$ implies that [11]

$$T_d < T_{d\,c} \approx 1.2 \left(\frac{Q}{f_B}\right)^{1/2} \left(\frac{100}{g(T_f)}\right)^{1/2} \left(\frac{10^{-10}}{\eta_B}\right)^{1/2} \left(\frac{m_\chi}{100\,\text{GeV}}\right) . \tag{4.12}$$

Since the decay temperature is estimated to be between 1MeV and 1 GeV [11], for reasonable values of Q and f_B (say $Q \gtrsim 0.1$) we expect that it is quite natural for this to be satisfied, especially for the case of a maximally charged condensate with $Q = f_B = 1$. However, if $T_d > T_{d\,c}$ then the LSP density is given by $n_{\text{limit}}(T_d)$, independently of the details of Q-ball decay,

$$n_{\text{limit}}(T_d) = \left(\frac{T_{d\,c}}{T_d}\right)^2 n_{LSP}(T_d) . \tag{4.13}$$

Although the direct connection between the baryon number and dark matter number density is lost, the neutralino dark matter will still be non-thermal in nature.

5. Non-spherical collapse and velocity effects

In the above discussion we have been considering values of f_B based on the spherically symmetric collapse of a single condensate lump and the evolution of an ensemble of static lumps being pulled apart from each other by the expansion of the Universe. However, it has been suggested that the effects of the velocity of condensate lumps after fragmentation [40] and the non-spherical collapse of condensate lumps [23, 40] may have very significant effects on the evolution of the ensemble of condensate lumps. In a recent numerical simulation [40] it was observed that after condensate fragmentation the lumps initially have a mildly relativistic velocity and can undergo collisions. (However, the lump velocity may depend upon the assumed form of the initial perturbation at $H \approx m$. For example, a standing wave perturbation may result in static lumps.) In addition, the large lumps which initially formed subsequently fragmented into $\pm$Q-balls. One question here is whether the formation of $\pm$Q-balls is due to the dynamics of condensate collapse or due to the linear growth of the perturbations of the charge density, as discussed in section 2? As well as mildly relativistic large Q-balls, smaller relativistic Q-balls were formed during condensate collapse. These, it is suggested, can effectively thermalize the distribution, allowing an analytical treatment [40]. The possibility of forming a significant number of small Q-balls in condensate fragmentation could have a significant effect on f_B. These could be emitted during the evolution of a non-spherical condensate lump, either due to being initially non-spherical after formation or due to the fusion of two colliding condensate lumps; for example, in [23] it was shown that in a $1+1$ dimensional simulation of Q-ball fusion, 7% of the charge was emitted in the form of very small Q-balls (or possibly scalar field waves, as the lattice simulation cannot distinguish these cases [23]). Since the rate of decay of Q-balls is proportional to Q^{-1} and the decay temperature $T_d \propto Q^{-1/2}$, it is possible that very small Q-balls could decay before the neutralinos freeze out of chemical equilibrium, $T_d > T_f$. In this case, since the neutralinos from Q-ball decay thermalize to become part of the thermal relic density and so are not related to the baryon number from the decaying Q-balls, the charge in small Q-balls is effectively removed from the Q-ball ensemble before the larger Q-balls decay, reducing f_B. So the tendency of non-spherical lumps and velocity on the formation and evolution of condensate lumps is to reduce f_B as compared with the idealized spherically symmetric static case.

6. Conclusions

We have considered a number of issues connected with the fragmentation of the Affleck-Dine condensate and the formation of late-decaying Q-balls, in particular the fraction of the baryon number in the Q-balls when they decay, f_B. For a non-maximally charged condensate we expect that either $\pm$Q-balls or Q-axitons will form, depending on the initial fluctuation of the phase of the condensate at $H \approx m$. We showed that in the limiting case where Q-axitons form numerical simulations of condensate fragmentation must be run for a sufficiently long time to ensure that absorption/re-emission of scalar field waves does not maintain the condensate lumps far from their classically stable Q-axiton state, so allowing the value of f_B to be extracted.

A major motivation for B-ball baryogenesis is a natural explaination for the baryon to dark matter number density ratio. In general, Q-ball formation with $f_B = 1$ from fragmentation of a MAX condensate and $\pm$Q-ball formation from a non-MAX condensate produces too many neutralinos per baryon number to be consistent with experimental bounds on MSSM neutralino masses if we wish to account for the baryon to dark matter particle number ratio via Q-ball decay. The only possibility is to have the limiting case of Q-axiton formation from a non-MAX condensate, with evolution of the Q-axitons to Q-balls before neutralinos freeze out of chemical equilibrium, such that $f_B \ll 1$ is possible. This is particularly possible if the Q-axiton evolves to a Q-ball via annihilations.

It is also possible that more complex features of AD condensate fragmentation which can only be studied numerically, such as the formation and evolution of non-spherically symmetric condensate lumps and collisons of moving lumps, might reduce the value of f_B, but it remains to be seen if these effects can reduce it sufficiently to accomodate MSSM neutralinos in the typical case of Q-ball formation from a MAX or $\pm$Q-ball formation from a non-MAX condensate.

In the typical case with $f_B/Q \gtrsim 1$ (rather than the limiting case of Q-axiton formation followed by evolution to Q-balls), the LSP neutralino mass from direct Q-ball decay must be less than about 20 GeV. This is too large to be consistent with MSSM neutralinos ($m_\chi > 32.2$ GeV experimentally), but is consistent with singlino dark matter in the NMSSM. Thus the B-ball baryogenesis explaination of the baryon to dark matter ratio strongly suggests that dark matter is non-thermal and that the LSP should be associated with an extension of the MSSM with an experimentally allowed light LSP, the NMSSM being the most obvious possibility. In addition, depending on details of the inflation model, there may be observable isocurvature density fluctuations due to the transfer of baryon isocurvature fluctuations to neutralinos via Q-ball decay [38]. Although all the evidence for B-ball baryogenesis is circumstantial, and each element could have an alternative explaination, if it turns out that the NMSSM is realized in nature with a light singlino LSP, if the LSP dark matter density is inconsistent with a thermal relic density and if isocurvature density perturbations are observed, then $d = 6$ B-ball baryogenesis would become a favoured scenario for the simultaneous origin of both the baryon number and dark matter density.

Finally, it should be emphasized that much work remains to be done to clarify the physics of unstable AD condensates and B-ball Baryogenesis. Given that $d = 6$

AD baryogenesis is a serious candidate for the origin of the baryon asymmetry in SUSY models and that late-decaying Q-balls could naturally account for the baryon to dark matter ratio, it is important to develop in detail the physics of AD condensate fragmentation in the early Universe.

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